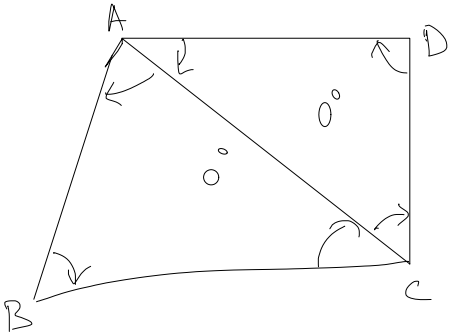
Q> Show that for any distinct points ABCD, we have,
$\angle ABC + \angle BCD + \angle CDA + \angle DAB = 0$

Ans:-



$\Rightarrow 0°$ overall

Q> Find all possible values of,
$x + \frac{4}{x}$ when $x \in \mathbb{R} \setminus \{0\}$

Ans:-

$x \in \mathbb{R}^+ \Rightarrow x + \frac{4}{x} \geq 2\sqrt{4}$

$x \in \mathbb{R}^- \Rightarrow -\left(x + \frac{4}{x}\right) \geq 2\sqrt{(x)\left(-\frac{4}{x}\right)} = 2\sqrt{4}$

$-x + \left(-\frac{4}{x}\right)$
$\in \mathbb{R}^+ \quad \in \mathbb{R}^+$

$\underbrace{\quad\quad}_{\in \mathbb{R}^+}$

$\Rightarrow \left(x + \frac{4}{x}\right) \leq -2\sqrt{4}$

Q> For $x < 0$ find the max value of $\frac{3x^2+12}{x}$.

Ans:-

$3x + \frac{12}{x} \Rightarrow -3x - \frac{12}{x} \geq 2\sqrt{36} = 12$

$-3x \in \mathbb{R}^+ \quad -\frac{12}{x} \in \mathbb{R}^+$

$\Rightarrow 3x + \frac{12}{x} \leq -12$

Q> $(x-2) + \frac{1}{x^2-16}$, $x > 0$, $x \neq 4$

$x - 2 > 0 \Rightarrow x > 2$
$x^2 - 16 > 0 \Rightarrow x > 4$

Ans:-

$x - 2 + \frac{1}{x^2-16} \geq 2\sqrt{\frac{x-2}{(x-4)(x+4)}}$

for $x > 4$

If $x \leq 2 \Rightarrow -(x-2) - \frac{1}{x^2-16} \geq 2\sqrt{\frac{x-2}{(x-4)(x+4)}}$

AM - GM ... will not work as one term is positive

If $x = -$,

If $x \in (2,4) \Rightarrow$ AM - GM will not work as one term is positive and one term is negative

Q) $f(x)$ is polynomial of degree 4, s.t. $f(1)=1, f(2)=2, f(3)=3$ $f(4), f(0)=1$. Find $f(5)$.

Ans:- $P(x) = f(x) - x$ $\quad P(x)=0$ for $x=1,2,3,4$.

$\Rightarrow$ degree of P is atmost 4.

$\Rightarrow P(x) = a(x-1)(x-2)(x-3)(x-4)$ $\quad P(0)=1$

$\Rightarrow 24a = 1$

$\Rightarrow a = \frac{1}{24}$

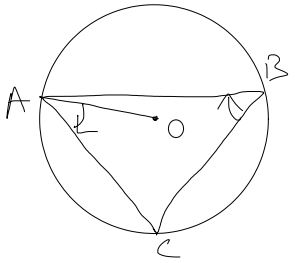
$\Rightarrow P(x) = \frac{1}{24}(x-1)(x-2)(x-3)(x-4)$

$= f(x) - x$

$\Rightarrow f(5) = \text{-----}$

HomeWork

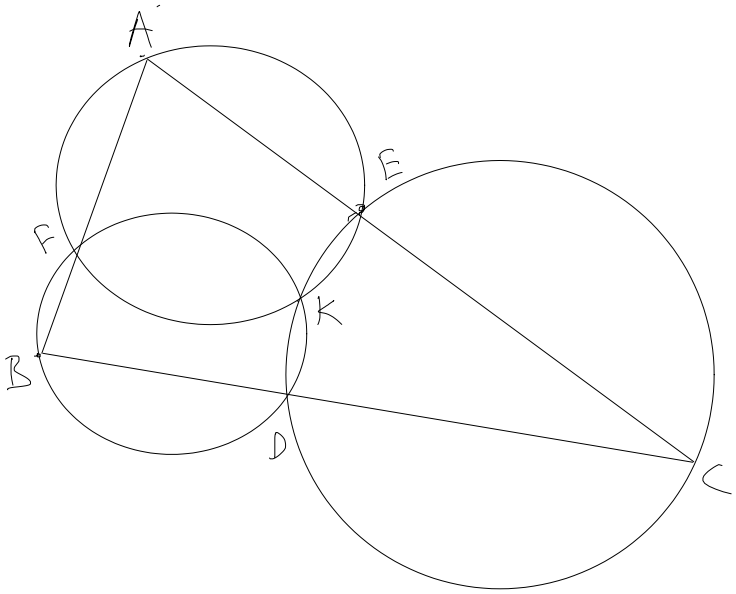
Q) Points A, B, C lie on a circle with centre O. Show that $\angle OAC = 90^\circ - \angle CBA$



HomeWork

Lemma:- (Miquel Point of a Triangle)

Points D, E, F lie on lines BC, CA, and AB of $\triangle ABC$, respc.
Then there exists a point K in on the three circles (AEF), (BFD), (CDE).



A, B, X, Y is cyclic
$\Leftrightarrow \measuredangle AXB = \measuredangle AYB$